# Improve time step size sensitivity in transient mechanical simulations

Jinglin Zheng and Xinhai Zhu
*Livermore Software Technology Corporation, Livermore, CA 94551, USA*

Time step size is one of the most important factors for explicit simulations. As a rule of thumb, we need to set the time step size to be small enough so that the update of a nodal solution is completed before the node is hit by the wave from neighboring nodes. Therefore, a proper time step size is dependent on the element size, as well as the sound speed of the material. However, in some cases we may get into a dilemma where the solution becomes unstable or divergent when we decrease the time step size for more accuracy. A simple example is shown below.

Consider a dog bone sample under a tensile test, as shown in Figure 1(a). The sample is subject to a uniaxial tension with the left end fixed and the right end being stretched at a constant rate of 0.02 m/s. Material type 260A is used for the model. A transient mechanical simulation is conducted to solve this simple elastic-plastic deformation problem. Figure 1(b) shows the time history of the axial force calculated by LS-DYNA® under three different time step sizes (*dt*): *dt* = 7e-6 s, *dt* = 7e-7 s and *dt* = 7e-8 s. The time history of effective plastic strain ($\varepsilon_p$) is plotted in Figure 1(c). A couple of obvious discontinuities can be observed from the blue curve in Figure 1(b), indicating that *dt* = 7e-6 s is not a proper time step size. As we reduce *dt* to 7e-7 s and further down to 7e-8 s, the curves do get smoother as expected. However, both axial force and effective plastic strain solutions start diverging, as shown by the red and green curves in Figures 1(b) and (c).

This phenomenon is not rare in real production simulations. In general, the numerical solution is expected to converge to the real one when the time step size decreases. But excessive reduction in time step size will result in the dominance of round-off error which can accumulate significantly and mess up the entire solution. In the specific case of Figures 1(b) and (c), the problem occurs at the iterative return mapping scheme which is used to find the incremental effective plastic strain ($\Delta\varepsilon_p$) at each time step. We can simplify the return mapping process as a search for the root to a non-linear equation $f(\Delta\varepsilon_p) = 0$. Usually a newton-type iteration procedure is employed to solve the equation. To determine if the trial solution is good enough, a fixed relative tolerance is applied as the convergence criteria, i.e., the searching process terminates if $|f(\Delta\varepsilon_p)| \leq tol \cdot Y(\Delta\varepsilon_p)$. Here *tol* is a fixed number (usually *tol* is set to be 0.001) and *Y* is the yield stress. Because $f(\Delta\varepsilon_p) = 0$ is solved numerically, we actually introduces a local error to the mechanical strain at every time step. The error spreads forward in the transient analysis. The precision of the solved $\Delta\varepsilon_p$ is up to the specified tolerance. Extremely small time steps could lead to small strain increment the error of which is likely to be ignored by the fixed tolerance criteria. Accordingly, the accumulated global error can become quite significant and eventually mess up the entire solution, as shown in Figures 1(b) and (c). A simple fix to this problem is to introduce a flexible tolerance which tightens up the accuracy requirement when the local error needs to be controlled for the purpose of convergence. A natural thought is to associate the convergence tolerance with time step size. However, as mentioned above, an applicable time step size varies with models significantly. Thus it is difficult to find one time step size that works for all the situations. On the other hand $\Delta\varepsilon_p$ might be a good choice because it reduces with time step size for the same model, and unlike the time step size, strain increment itself is a relative measure. The only problem is that $\Delta\varepsilon_p$ is unknown (we are actually trying to solve $\Delta\varepsilon_p$ with the return mapping algorithm). However, we do have a very good candidate to serve as $\Delta\varepsilon_p$ in an iterative scheme, which is the guess of $\Delta\varepsilon_p$.

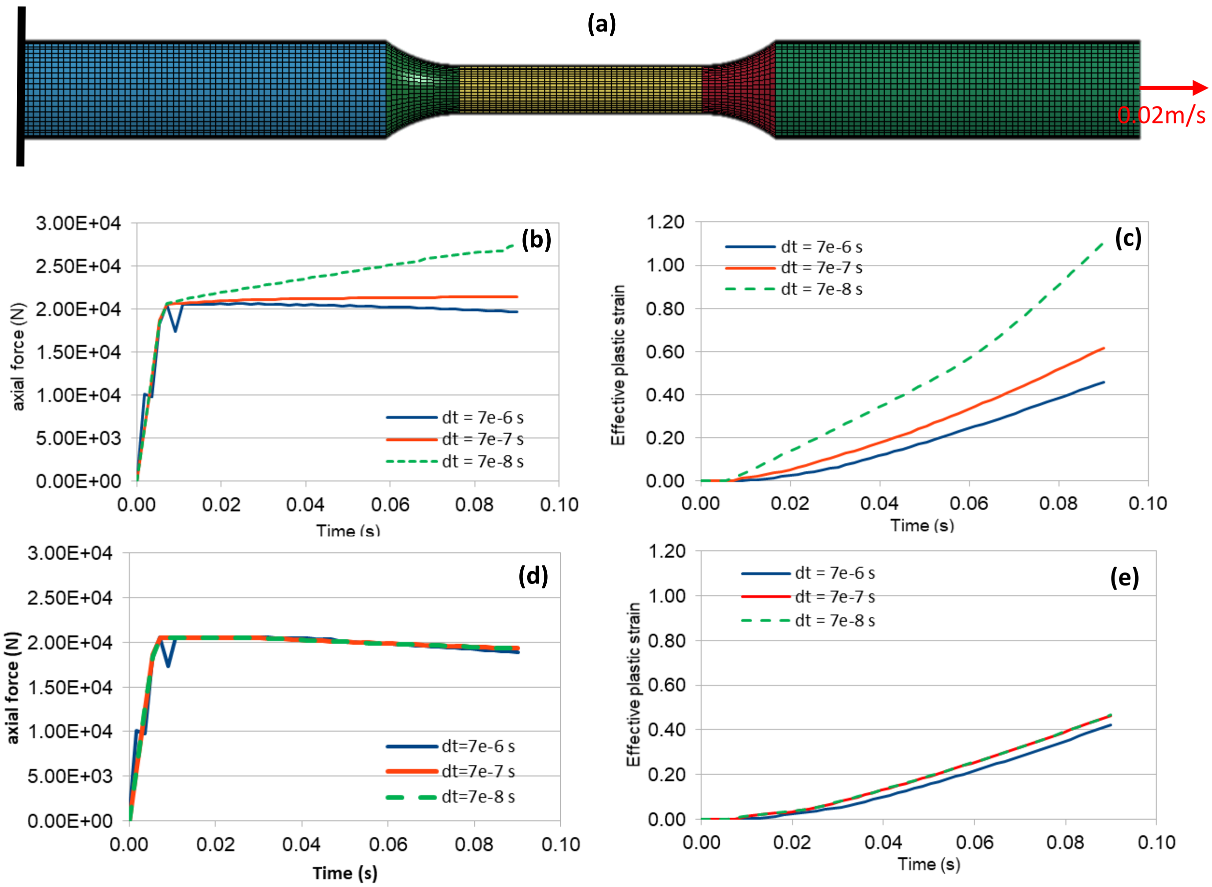


Figure 1 Example uniaxial tension problem (a) Tensile test on a dog bone sample (b) Before fix: time history of axial force under three different time step settings: *dt* = 7e-6 s, 7e-7 s, 7e-8 s (c) Before fix: time history of effective plastic strain under three different time step settings: *dt* = 7e-6 s, 7e-7 s, 7e-8 s (d) After fix: time history of axial force under three different time step settings: *dt* = 7e-6 s, 7e-7 s, 7e-8 s (e) After fix: time history of effective plastic strain under three different time step settings: *dt* = 7e-6 s, 7e-7 s, 7e-8 s

Figures. 1(d) and (e) show the results when the flexible tolerance is applied in return mapping. As shown in Figure 1(d), time history of the axial force becomes smooth when *dt* is reduced to 7e-7 s and below. In the meantime, the resultant axial force converges successfully as *dt* decreases, in contrast to the diverging trend in Figure 1(b). Similarly, the effective plastic strain curves in Figure 1(e) also shows a nice trend of convergence as *dt* reduces toward 7e-8 s. The conflict between time step size and numerical stability is thus resolved.